Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

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0123456789

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/05**

Paper 5 Investigation (Core) **For examination from 2025**

SPECIMEN PAPER B **1 hour 15 minutes**

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 40.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Any blank pages are indicated.

INVESTIGATION ADDING SQUARE NUMBERS

In this investigation, you will look at adding two or more square numbers to make another square number.
All numbers are positive integers.

1 Complete the list of the first six square numbers.

$1^2 = 1$ $2^2 =$ .... $3^2 = 9$ $4^2 =$ .... $5^2 =$ .... $6^2 = 36$ [1]

2 **(a)** Work out.

(i) 9^2

.... [1]

(ii) 40^2

.... [1]

(b) Show that $9^2 + 40^2 = 41^2$.

[2]

3

When $a^2 + b^2 = c^2$ then (a, b, c) is called a 3-square set.
a, b and c are positive integers.

<u>Example</u>

In **Question 2(b)**, $a = 9$, $b = 40$ and $c = 41$.
$9^2 + 40^2 = 41^2$, so (9, 40, 41) is a 3-square set.

When $a^2 + b^2 = c^2$ then $c = \sqrt{a^2 + b^2}$.

Use this formula and any patterns you notice to complete the table of 3-square sets on the next page.

a	b	c
3	4	5
5	12	13
7	24	25
9	40	41
11	60	
13	84	85
	112	113
	144	
19		181
21		221
25	312	313

[6]

4 When $a^2 + b^2 + c^2 = d^2$ then (a, b, c, d) is called a 4-square set.
It is possible to make a 4-square set using two rows in the table.

<u>Example</u> From the table row two $5^2 + 12^2 = 13^2$
row six $13^2 + 84^2 = 85^2$

Replace 13^2 in the second equation with $5^2 + 12^2$ from the first equation: $5^2 + 12^2 + 84^2 = 85^2$.

So (5, 12, 84, 85) is a 4-square set.

Use the same method with rows from the table to find two more 4-square sets.

(......... , ... , ... , ...) and (......... , ... , ... , ...) [3]

5 **(a)** Show that (7, 14, 14, 21) is a 4-square set.

[2]

(b) k is any positive integer greater than 1.
(ka, kb, kc, kd) is a 4-square set.
So $(ka)^2 + (kb)^2 + (kc)^2 = (kd)^2$.

By removing the brackets in the equation, show that (a, b, c, d) must also be a 4-square set.

[2]

(c) The numbers in the 4-square set (7, 14, 14, 21) have a common factor greater than 1.

(i) Write down this common factor.

.... [1]

(ii) Find a 4-square set where (a, b, c, d) do not have a common factor greater than 1.
Start with (7, 14, 14, 21) and use **part (b)** to help you.

(......... , ... , ... , ...) [2]

(d) (12, 36, 54, 66) is a 4-square set.

(i) Find the highest common factor (HCF) of 12, 36, 54 and 66.

.... [1]

(ii) Use **part (i)** to find a 4-square set in which the numbers do not have a common factor greater than 1.

(......... , ... , ... , ...) [1]

(iii) Show that your answer to **part (ii)** is a 4-square set.

[2]

6 Here is a method for finding a 4-square set (a, b, c, d).

Choose two positive integers a and b with a less than b.

Then use $c = \dfrac{a^2 + b^2 - 1}{2}$ and $d = \dfrac{a^2 + b^2 + 1}{2}$ to make the 4-square set (a, b, c, d).

(a) Use this method to find a 4-square set when

(i) $a = 2$ and $b = 3$

(2, 3, ... , ...) [3]

(ii) $a = 2$ and $d = 43$.

(2, ... , ... , 43) [3]

(b) **(i)** Complete the table of 4-square sets that start with 2.
Use your answers to **part (a)** and any patterns you notice.

a	b	c	d
2	3		
2	5	14	15
2	7	26	27
2			43
2			

[3]

(ii) Write down an equation connecting c and d.

.... [1]

(c) When a and b are both even then $c = \frac{a^2 + b^2 - 1}{2}$ and $d = \frac{a^2 + b^2 + 1}{2}$ do not give a 4-square set.

Give an example to show that this statement is correct.

[2]

(d) When a and b are both odd there are no 4-square sets.

In a 4-square set, $d = 23$.

(i) Show that $a^2 + b^2 = 45$.

[1]

(ii) Find a 4-square set when $d = 23$.

(......... , ... , ... , 23) [2]

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