Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/01**

Paper 1 Further Pure Mathematics **For examination from 2020**

MARK SCHEME

Maximum Mark: 120

Specimen

This specimen paper has been updated for assessments from 2020. The specimen questions and mark schemes remain the same. The layout and wording of the front covers have been updated to reflect the new Cambridge International branding and to make instructions clearer for candidates.

This document has **16** pages. Blank pages are indicated.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

The following abbreviations may be used in a mark scheme:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
aef Any equivalent form
art Answers rounding to
cwo Correct working only (emphasising that there must be no incorrect working in the solution)
ft Follow through from previous error is allowed
o.e. Or equivalent
D Dependent mark (dependent on an earlier mark in the scheme)

Question	Answer	Marks	Notes
1	$\sum_{r=1}^{n}(r^2 - r + 1) = \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$ Splitting summation and use of given results	M1	
	$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) + n$ 1st B1 for Σr^2; 2nd B1 for Σr & $\Sigma 1 = n$	B1B1	
	$= \frac{1}{3}n(n^2+2)$ legitimately **AG**	A1	
		4	

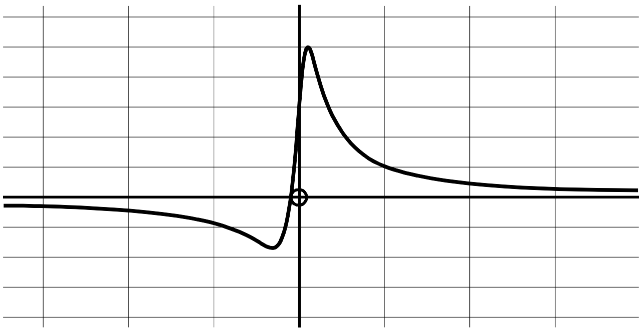
Question	Answer	Marks	Notes
2	$A = k\int(\sin\theta + \cos\theta)^2 \mathrm{d}\theta$ including squaring attempt; ignore limits and $k \neq \frac{1}{2}$	M1	
	$= \frac{1}{2}\int(1 + \sin 2\theta)\mathrm{d}\theta$ B1 for use of the double-angle formula **OR** integration of $\sin\theta\cos\theta$ as $k\sin^2\theta$ or $k\cos^2\theta$	B1	
	$= \frac{1}{2}\left[\theta - \frac{1}{2}\cos 2\theta\right]_0^{\frac{\pi}{2}}$ **ft** (constants only) in the integration; MUST be 2 separate terms	A1	A1ft
	$\frac{1}{4}\pi + \frac{1}{2}$	A1	
		4	

Question	Answer	Marks	Notes
3(a)	Method for Sarrus' Rule, or expanding by R_1 for example	M1	
	Det = $11k - 66$	A1	
		2	
3(b)	$k = 6$ **ft** from their Det = 0	B1	B1ft
		1	
3(c)	**EITHER** e.g. ③ – 6 × ① ⇒ $z = 7$ e.g. ③ + 2 × ② ⇒ $22y + 23z = 73 \Rightarrow 22y = -88 \Rightarrow y = -4$ M1 for a complete solution strategy	M1	
	e.g. $x = 4 - 2y - z = 5$ A1 for first correct	A1	
	$x = 5, y = -4, z = 7$ A1 for all 3 correct	A1	
	OR $\frac{1}{11}\begin{pmatrix} -61 & -2 & 11 \\ 69 & 1 & -11 \\ -66 & 0 & 11 \end{pmatrix}\begin{pmatrix} 4 \\ 21 \\ 31 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 7 \end{pmatrix}$ M1 for complete method	M1	
	B1 for correct inverse of the matrix of coefficients	B1	
	A1 for correct answer	A1	
	Available marks	**3**	

Question	Answer	Marks	Notes
4(a)	$y=(\sinh x)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx}=\frac{1}{2}(\sinh x)^{-\frac{1}{2}}.\cosh x$ **OR** $y^2=\sinh x \Rightarrow 2y\frac{dy}{dx}=\cosh x$	M1A1	
	$=\frac{\sqrt{1+y^4}}{2y}$	A1	
		3	
4(b)	$\int \frac{2y}{\sqrt{1+y^4}}dy=\int 1.dx$ By separating variables in **(a)**'s answer	M1	
	$\Rightarrow x=\int \frac{2y}{\sqrt{1+y^4}}dy$	A1	
	But $x=\sinh^{-1}y^2$ so $\int \frac{2t}{\sqrt{1+t^4}}dx=\sinh^{-1}(t^2)+C$ condone missing " $+ C$ "	A1	
	OR **ALTERNATE SOLUTION 1** Set $t^2=\sinh\theta$, $2t\,dt=\cosh\theta\, d\theta$ M1 for full substitution	M1	
	$\int \frac{2t}{\sqrt{1+t^4}}dt=\int \frac{\cosh\theta}{\sqrt{1+\sinh^2\theta}}d\theta$	A1	
	$\int 1.d\theta=\theta=\sinh^{-1}(t^2)+C$	A1	
	OR **ALTERNATE SOLUTION 2** Set $t^2=\tan\theta$, $2t\,dt=\sec^2\theta\, d\theta$ M1 for full substitution	M1	
	$\int \frac{2t}{\sqrt{1+t^4}}dt=\int \frac{\sec^2\theta}{\sqrt{1+\tan^2\theta}}d\theta$ $=\int \sec\theta.d\theta$	A1	
	$=\ln\lvert\sec\theta+\tan\theta\rvert+C=\ln\left\lvert t^2+\sqrt{1+t^4}\right\rvert+C$	A1	
	Available marks	**3**	

Question	Answer	Marks	Notes
5	For $n = 1$, LHS = $\frac{2}{3} \times \frac{2}{7} = \frac{4}{21}$ and RHS = $\frac{1}{3} - \frac{1}{7} = \frac{4}{21}$ (hence result is true for $n = 1$)	B1	
	Assuming that $\sum_{r=1}^{k} \left(\frac{2}{4r-1}\right)\left(\frac{2}{4r+3}\right) = \frac{1}{3} - \frac{1}{4k+3}$ Stated explicitly, or induction hypothesis made entirely clear from later working (do not accept "statement true for $n = k$" unless it is made clear)	M1	
	$\sum_{r=1}^{k+1} \left(\frac{2}{4r-1}\right)\left(\frac{2}{4r+3}\right) = \frac{1}{3} - \frac{1}{4k+3} + \frac{2}{4k+3}.\frac{2}{4k+7}$ Adding $(k + 1)$th term;	M1	
	correct	A1	
	$= \frac{1}{3} - \left(\frac{1}{4k+3} - \frac{2}{4k+3}.\frac{2}{4k+7}\right)$ Separating off the algebraic terms	M1	
	$= \frac{1}{3} - \frac{1}{4k+3}\left(\frac{4k+7}{4k+7} - \frac{4}{4k+7}\right)$ Factorisation and common denominator	M1	
	$= \frac{1}{3} - \frac{1}{4(k+1)+3}$ Correct answer *demonstrated* to be of the right form	A1	
	Case 1 true and Case $n = k$ true $\Rightarrow$ Case $n = k + 1$ true gives the result by induction (induction reasoning must be clear)	A1	
		8	

Question	Answer	Marks	Notes
6(a)	$y = \frac{x+1}{x^2+3} \Rightarrow y.x^2 - x + (3y - 1) = 0$ Creating a quadratic in x	M1	
	For real x, $1 - 4y(3y - 1) \geqslant 0$ Considering the discriminant	M1	
	$12y^2 - 4y - 1 \leqslant 0$ Creating a quadratic **inequality**	M1	
	For real x, $(6y + 1)(2y - 1) \leqslant 0$ Factorising/solving a 3-term quadratic	M1	
	$-\frac{1}{6} \leqslant y \leqslant \frac{1}{2}$ CAO	A1	
		5	
6(b)	$y = \frac{1}{2}$ substituted back $\Rightarrow \frac{1}{2}(x^2 - 2x + 1) = 0 \Rightarrow x = 1$ $\left[y = \frac{1}{2}\right]$	M1A1	
	$y = -\frac{1}{6}$ substituted back $\Rightarrow -\frac{1}{6}(x^2 + 6x + 9) = 0 \Rightarrow x = -3$ $\left[y = -\frac{1}{6}\right]$	M1A1	
		4	

Question	Answer	Marks	Notes
6(c)	 Turning points **ft** correct	B1	B1ft
	B1 for (–1, 0), B1 for (0, $\frac{1}{3}$) noted or marked	B1B1	
	Correct shape in **ft** range	B1	B1ft
		4	

Question	Answer	Marks	Notes
7(a)	Substituting $x = 1$, $f(1) = 2$ and $f'(1) = 3$ into (*) $\Rightarrow$ $f''(1) = 5$	M1A1	
		2	
7(b)	Product Rule used twice; at least one bracket correct	M1	
	$\{x^2 f'''(x) + 2xf''(x)\} + \{(2x-1)f''(x) + 2f'(x)\} - 2f'(x) = 3e^{x-1}$	A1	
	Substituting $x = 1$, $f'(1) = 3$ and $f''(1) = 5$ into this $\Rightarrow f'''(1) = -12$ **ft** their $f''(1)$	M1A1	M1A1ft
		4	
7(c)	$f(x) = f(1) + f'(1)(x-1) + \frac{1}{2}f''(1)(x-1)^2 + \frac{1}{6}f'''(1)(x-1)^3 + \ldots$ Use of the Taylor series	M1	
	$= \ 2 + 3(x-1) + \frac{5}{2}(x-1)^2 - 2(x-1)^3 + \ldots$ 1st two terms CAO; 2nd two terms **ft (a)** & **(b)**’s answers	A1A1	A1A1ft
		3	
7(d)	Substituting $x = 1.1$ $\Rightarrow$ $f(1.1) \approx 2.323$ to 3d.p. CAO	M1A1	
		2	

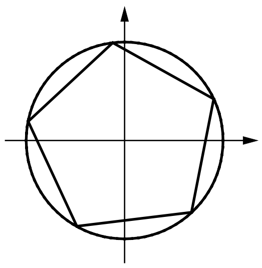
Question	Answer	Marks	Notes
8(a)	Good attempt to multiply 2 matrices of the appropriate form: $\begin{pmatrix} p & p \\ p & p \end{pmatrix}\begin{pmatrix} q & q \\ q & q \end{pmatrix}$	M1	
	"Closure" noted or implied by correct product matrix = $\begin{pmatrix} 2pq & 2pq \\ 2pq & 2pq \end{pmatrix} \in S$	A1	
	Statement that $\times_M$ known to be associative **OR** **Alternative** $[(p)(q)]\,(r) = (p)\,[(q)(r)] = (4pqr)$ shown	B1	
	Identity is $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} (\in S)$	B1	
	$\begin{pmatrix} p & p \\ p & p \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{4p} & \frac{1}{4p} \\ \frac{1}{4p} & \frac{1}{4p} \end{pmatrix} (\in S \text{ as } p \neq 0)$ … and $(S, \times_M)$ is a group since all four group axioms are satisfied	B1	
		5	
8(b)	Attempt to look for a self-inverse element; i.e. solving $p = \dfrac{1}{4p}$ using their $(p)^{-1}$ and **E**	M1	
	$p = -\frac{1}{2}$ and noting that $H = \{\mathbf{E}, \mathbf{A}\}$ where $\mathbf{E} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, $\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$	A1	
	Looking for $\{\mathbf{E}, \mathbf{B}, \mathbf{B}^2\}$ where $\mathbf{B}^3 = \mathbf{E}$; i.e. solving $(4p^3) = \frac{1}{2}$	M1	
	Explaining carefully that $p^3 = \frac{1}{8} \Leftrightarrow p = \frac{1}{2}$ and no such $\mathbf{B}$ $(\neq \mathbf{E})$ exists	A1	
		4	

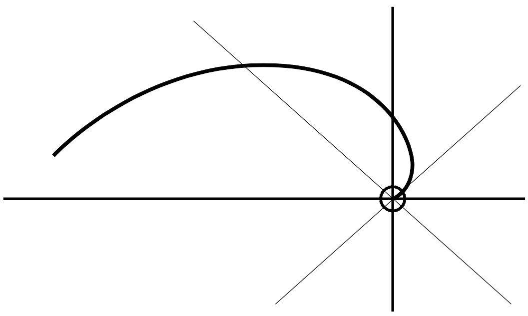
Question	Answer	Marks	Notes
9(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}+y=3xy^4$ is a *Bernoulli (differential) equation* $u=\frac{1}{y^3}\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=-\frac{3}{y^4}\times\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
	Then $\frac{\mathrm{d}y}{\mathrm{d}x}+y=3xy^4$ becomes $-\frac{3}{y^4}\times\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{3}{y^3}=-9x\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}-3u=-9x$ **AG**	M1A1	
		3	
9(b)	**METHOD 1** Integrating factor is $\mathrm{e}^{\int -3\mathrm{d}x}=\mathrm{e}^{-3x}$	M1A1	
	$\Rightarrow u\mathrm{e}^{-3x}=\int -9x\mathrm{e}^{-3x}\mathrm{d}x$	M1	
	$=3x\mathrm{e}^{-3x}-\int 3\mathrm{e}^{-3x}\mathrm{d}x$ Use of "parts"	M1	
	$=(3x+1)\,\mathrm{e}^{-3x}+C$	A1	
	General solution is $u=3x+1+C\mathrm{e}^{3x}$ **ft**	B1	B1ft
	$\Rightarrow y^3=\frac{1}{3x+1+C\mathrm{e}^{3x}}$ **ft**	B1	B1ft
	Using $x=0, y=\frac{1}{2}$ to find C	M1	
	$C=7$ or $y^3=\frac{1}{3x+1+7\mathrm{e}^{3x}}$	A1	
	OR **METHOD 2** Auxiliary equation $m-3=0 \Rightarrow u_C=A\mathrm{e}^{3x}$ is the complementary function	M1A1	
	For particular integral try $u_P=ax+b$, $u_P{}'=a$	M1	
	Substituting $u_P=ax+b$ and $u_P{}'=a$ into the d.e. and comparing terms	M1	
	$a-3ax-3b=-9x \Rightarrow a=3, b=1$ i.e. $u_P=3x+1$	A1	
	General solution is $u=3x+1+A\mathrm{e}^{3x}$ **ft** particular integral + complementary function provided particular integral has no arbitrary constants and complementary function has one	B1	B1ft
	$\Rightarrow y^3=\frac{1}{3x+1+A\mathrm{e}^{3x}}$ **ft**	B1	B1ft
	Using $x=0, y=\frac{1}{2}$ to find A	M1	
	$A=7$ or $y^3=\frac{1}{3x+1+7\mathrm{e}^{3x}}$	A1	
	Available marks	**9**	

Question	Answer	Marks	Notes
10(a)	Substituting $\begin{pmatrix} 1+3\lambda \\ -3+4\lambda \\ 2+6\lambda \end{pmatrix}$ into plane equation; i.e. $\begin{pmatrix} 1+3\lambda \\ -3+4\lambda \\ 2+6\lambda \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix} = k$ **OR** any point on line (since "given")	M1	
	$k = 2 + 6\lambda + 18 - 24\lambda + 6 + 18\lambda = 26$	A1	
		2	
10(b)	Working with vector $\begin{pmatrix} 10+2m \\ 2-6m \\ 3m-43 \end{pmatrix}$.	B1	
	Substituting into the plane equation: $\begin{pmatrix} 10+2m \\ 2-6m \\ 3m-43 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix} = k$	M1	
	Solving a linear equation in m: $20 + 4m - 12 + 36m + 9m - 129 = 26$	M1	
	$m = 3 \Rightarrow Q = (16, -16, -34)$	A1	
	Shortest distance is $\lvert m\rvert \left\lVert \begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix} \right\rVert = 21$ or $PQ = \sqrt{6^2 + 18^2 + 9^2} = 21$	A1	
		5	

Question	Answer	Marks	Notes
10(c)	Finding 3 points in the plane: e.g. $A(1, -3, 2)$, $B(4, 1, 8)$, $C(10, 2, -43)$ **OR** B1 for one vector in the plane	M1	
	Then 2 vectors in (// to) plane: e.g. $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}, \overrightarrow{AC} \begin{pmatrix} 9 \\ 5 \\ -45 \end{pmatrix}, \overrightarrow{BC} \begin{pmatrix} 6 \\ 1 \\ -51 \end{pmatrix}$ **OR** B1 for another vector in the plane	M1	
	Vector product of any two of these to get normal to plane: $\begin{pmatrix} 10 \\ -9 \\ 1 \end{pmatrix}$ (any non-zero multiple)	M1A1	
	$d = \begin{pmatrix} 10 \\ -9 \\ 1 \end{pmatrix} \bullet (\text{any position vector}) = \begin{pmatrix} 10 \\ -9 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ e.g. = 39	M1	
	$\Rightarrow 10x - 9y + z = 39$ CAO (aef)	A1	
	OR **ALTERNATE SOLUTION** $ax + by + cz = d$ contains $\begin{pmatrix} 1+3\lambda \\ -3+4\lambda \\ 2+6\lambda \end{pmatrix}$ and $\begin{pmatrix} 10 \\ 2 \\ -43 \end{pmatrix}$ ... so $a + 3a\lambda + 4b\lambda - 3b + 2c + 6c\lambda = d$ and $10a + 2b - 43c = d$	M1B1	
	Then $a - 3b + 2c = d$ and $3a + 4b + 6c = 0$ (λ terms) i.e. equating terms	M1	
	Eliminating (e.g.) c from 1st two equations $\Rightarrow 9a + 10b = 0$	M1	
	Choosing $a = 10, b = -9 \Rightarrow c = 1$ and $d = 39$ i.e. $10x - 9y + z = 39$ CAO	M1A1	
	Available marks	**6**	

Question	Answer	Marks	Notes
11(a)	$\sin 5\theta = \text{Im}(\cos 5\theta + \text{i}\sin 5\theta) = \text{Im}(c + \text{i}s)^5$	M1	
	$(c + \text{i}s)^5 = c^5 + 5c^4\,\text{i}s + 10c^3\text{i}^2s^2 + 10c^2\text{i}^3s^3 + 5c\text{i}^4s^4 + \text{i}^5s^5$	M1	
	Im part = $s\,(5c^4 - 10c^2s^2 + s^4)$	A1	
	$= s(5(1 - s^2)^2 - 10(1 - s^2)s^2 + s^4)$	M1	
	$= s(16s^4 - 20s^2 + 5)$ legitimately **AG**	A1	
	$\sin 5\theta = 0 \Rightarrow 5\theta = 0, \pm\pi, \pm 2\pi$, etc. $\Rightarrow \theta = 0, \pm\frac{\pi}{5}, \pm\frac{2\pi}{5}$, etc.	M1	
	$s^2 = \frac{20 \pm \sqrt{80}}{32} = \frac{5 \pm \sqrt{5}}{8}$	M1	
	Since $\frac{2\pi}{5}$ is acute and sine is an increasing function for acute angles, $s = \sin\frac{2\pi}{5} = \sqrt{\frac{5+\sqrt{5}}{8}}$ with explanation (allow "largest positive root wanted")	A1	
		8	
11(b)	$\lvert\omega\rvert = 32$	B1	
	for use of $\tan^{-1}(\sqrt{3})$	M1	
	for arg $\omega = \frac{2\pi}{3}$	A1	
		3	

Question	Answer	Marks	Notes
11(c)(i)	$z^5 = \left(32, \frac{-10\pi}{3}\right), \left(32, \frac{-4\pi}{3}\right), \left(32, \frac{2\pi}{3}\right), \left(32, \frac{8\pi}{3}\right), \left(32, \frac{14\pi}{3}\right)$ for use of modulus & argument	M1	
	for considering at least two others $\pm 2n\pi$	M1	
	$\Rightarrow z = \left(2, \frac{-2\pi}{3}\right), \left(2, \frac{-4\pi}{15}\right), \left(2, \frac{2\pi}{15}\right), \left(2, \frac{8\pi}{15}\right), \left(2, \frac{14\pi}{15}\right)$ **ft** $\sqrt[5]{\text{mod}}$	B1	B1ft
	their arg/5	M1	
	all correct	A1	
		5	
11(c)(ii)	 for 5 points on circle, centre *O*, radius 2, equally spread out	B1	
	$\text{Area} = 5 \times \frac{1}{2} \times 2 \times 2 \times \sin\frac{2\pi}{5}$	M1	
	$= 10\sqrt{\frac{5+\sqrt{5}}{8}}$ or exact equivalent	A1	
		3	

Question	Answer	Marks	Notes
12(a)	$I_n = \int_0^3 x^{n-1}(x\sqrt{16+x^2})\,dx$ Correct splitting *and* use of parts	M1	
	$= \left[x^{n-1}.\frac{(16+x^2)^{\frac{3}{2}}}{3} \right]_0^3 - \int_0^3 (n-1)x^{n-2}\frac{(16+x^2)^{\frac{3}{2}}}{3}\,dx$	A1	
	$= 3^{n-2}.125 - \left(\frac{n-1}{3}\right)\int_0^3 x^{n-2}(16+x^2)\sqrt{16+x^2}\,dx$ Method to get 2nd integral of correct form	M1	
	$= 3^{n-2}.125 - \left(\frac{n-1}{3}\right)\{16I_{n-2} + I_n\}$ [i.e. reverting to *I*'s in 2nd integral]	M1	
	$\Rightarrow 3\,I_n = 3^{n-1}.125 - 16(n-1)\,I_{n-2} - (n-1)\,I_n$ Collecting up I_ns	M1	
	$(n+2)\,I_n = 125 \times 3^{n-1} - 16(n-1)\,I_{n-2}$ **AG**	A1	
		6	
12(b)(i)	 Spiral (with *r* increasing)	B1	
	From *O* to just short of $\theta = \pi$	B1	
		2	
12(b)(ii)	$r = \frac{1}{4}\theta^4 \Rightarrow \frac{dr}{d\theta} = \theta^3$ and $r^2 + \left(\frac{dr}{d\theta}\right)^2 = \frac{1}{16}\theta^8 + \theta^6$	M1A1	
	$L = \int_0^3 \frac{1}{4}\theta^3\sqrt{16+\theta^2}\left(= \frac{1}{4}I_3\right)$	M1A1	
	Now $I_1 = \left[\frac{1}{3}(16+x^2)^{\frac{3}{2}}\right]_0^3 = \frac{61}{3}$	B1	
	and $5I_3 = 125 \times 9 - 16 \times 2\left(\frac{61}{3}\right) = \frac{1423}{3}$ or $474\frac{1}{3}$ Use of given reduction formula	M1	
	so that $L = \frac{1}{20} \times \frac{1423}{3} = \frac{1423}{60}$ or $23\frac{43}{60}$ **ft** only from suitable $k\,I_3$	A1	A1ft
		7	

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