Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/01**

Paper 1 Further Pure Mathematics **For examination from 2020**

SPECIMEN PAPER **3 hours**

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You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS
- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 120.
- The number of marks for each question or part question is shown in brackets [ ].

 [Turn over

1 Using any standard results given in the List of Formulae (MF20), show that

$$\sum_{r=1}^{n}(r^2 - r + 1) = \tfrac{1}{3}n(n^2 + 2)$$

for all positive integers n. [4]

2 A curve has polar equation $r = \sin\theta + \cos\theta$. Find the area enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{2}\pi$. [4]

3 **(a)** Evaluate, in terms of k, the determinant of the matrix $\begin{pmatrix} 1 & 2 & 1 \\ -3 & 5 & 8 \\ 6 & 12 & k \end{pmatrix}$. [2]

Three planes have equations $x + 2y + z = 4$, $-3x + 5y + 8z = 21$ and $6x + 12y + kz = 31$.

(b) State the value of k for which these three planes do not meet at a single point. [1]

(c) Find the coordinates of the point of intersection of the three planes when $k = 7$. [3]

4 **(a)** Given that $y = \sqrt{\sinh x}$ for $x \geqslant 0$, express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y only. [3]

(b) Hence or otherwise find $\int \frac{2t}{\sqrt{1 + t^4}}\,\mathrm{d}t$. [3]

5 Use induction to prove that $\sum_{r=1}^{n}\left(\frac{2}{4r-1}\right)\left(\frac{2}{4r+3}\right) = \frac{1}{3} - \frac{1}{4n+3}$ for all positive integers n. [8]

6 The curve C has equation $y = \frac{x+1}{x^2+3}$.

(a) By considering a suitable quadratic equation in x, find the set of possible values of y for points on C. [5]

(b) Deduce the coordinates of the turning points on C. [4]

(c) Sketch C. [4]

7 The function f satisfies the differential equation

$$x^2 \mathrm{f}''(x) + (2x - 1)\mathrm{f}'(x) - 2\mathrm{f}(x) = 3\mathrm{e}^{x-1} + 1, \qquad (*)$$

and the conditions $\mathrm{f}(1) = 2$, $\mathrm{f}'(1) = 3$.

(a) Determine $\mathrm{f}''(1)$. [2]

(b) Differentiate $(*)$ with respect to x and hence evaluate $\mathrm{f}'''(1)$. [4]

(c) Hence determine the Taylor series approximation for $\mathrm{f}(x)$ about $x = 1$, up to and including the term in $(x-1)^3$. [3]

(d) Deduce, to 3 decimal places, an approximation for $\mathrm{f}(1.1)$. [2]

8 Consider the set S of all matrices of the form $\begin{pmatrix} p & p \\ p & p \end{pmatrix}$, where p is a non-zero rational number.

(a) Show that S, under the operation of matrix multiplication, forms a group, G. (You may assume that matrix multiplication is associative.) [5]

(b) Find a subgroup of G of order 2 and show that G contains no subgroups of order 3. [4]

9 **(a)** Show that the substitution $u = \dfrac{1}{y^3}$ transforms the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = 3xy^4$ into

$$\frac{\mathrm{d}u}{\mathrm{d}x} - 3u = -9x. \qquad [3]$$

(b) Solve the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = 3xy^4$, given that $y = \frac{1}{2}$ when $x = 0$. Give your answer in the form $y^3 = \mathrm{f}(x)$. [9]

10 The line L has equation $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$ and the plane Π has equation $\mathbf{r}.\begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix} = k$.

(a) Given that L lies in Π, determine the value of k. [2]

(b) Find the coordinates of the point, Q, in Π which is closest to $P(10, 2, -43)$. Deduce the shortest distance from P to Π. [5]

(c) Find, in the form $ax + by + cz = d$, where a, b, c and d are integers, an equation for the plane which contains both L and P. [6]

11 **(a)** Use de Moivre's theorem to prove that $\sin 5\theta \equiv s(16s^4 - 20s^2 + 5)$, where $s = \sin\theta$, and deduce that $\sin\frac{2\pi}{5} = \sqrt{\frac{5+\sqrt{5}}{8}}$. [8]

The complex number $\omega = 16(-1 + \mathrm{i}\sqrt{3})$.

(b) State the value of $|\omega|$ and find arg ω as a rational multiple of π. [3]

(c) **(i)** Determine the five roots of the equation $z^5 = \omega$, giving your answers in the form (r, θ), where $r > 0$ and $-\pi < \theta \leqslant \pi$. [5]

(ii) These five roots are represented in the complex plane by the points A, B, C, D and E. Show these points on an Argand diagram, and find the area of the pentagon $ABCDE$ in an exact surd form. [3]

12 **(a)** Let $I_n = \int_0^3 x^n \sqrt{16 + x^2}\, dx$, for $n \geqslant 0$. Show that, for $n \geqslant 2$,

$$(n + 2)I_n = 125 \times 3^{n-1} - 16(n - 1)I_{n-2}.$$ [6]

(b) A curve has polar equation $r = \frac{1}{4}\theta^4$ for $0 \leqslant \theta \leqslant 3$.

(i) Sketch this curve. [2]

(ii) Find the exact length of the curve. [7]